



# Constrained DFT to mimick thermalized photo-excited carriers

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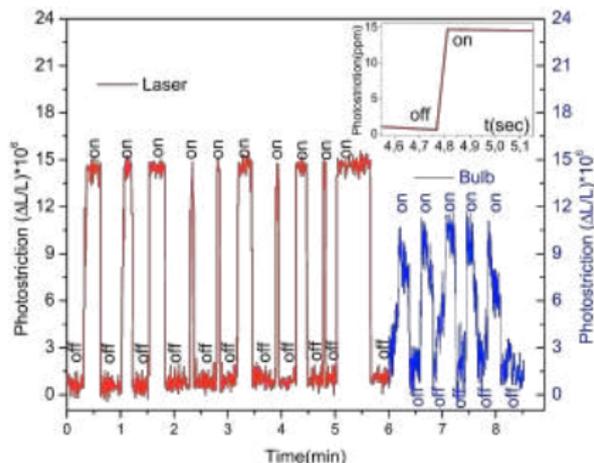
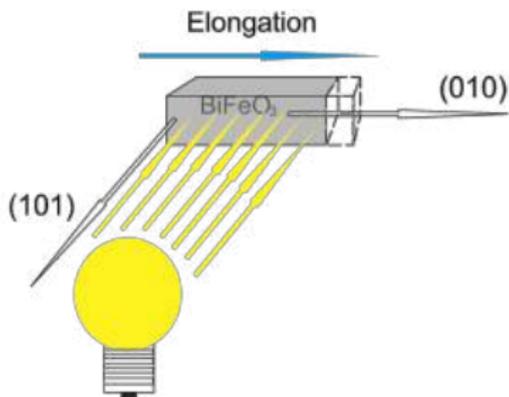
# Outline

- 1 Introduction
- 2 Implementation
- 3 Photo-induced phase transitions



# Photostriction in ferroelectrics

- Ferroelectric = functional material with switchable electrical polarization
- Photostriction = change of shape under illumination.
- BFO: longitudinal distortion of  $\approx 1.5 \times 10^{-3} \%^{-1}$ .

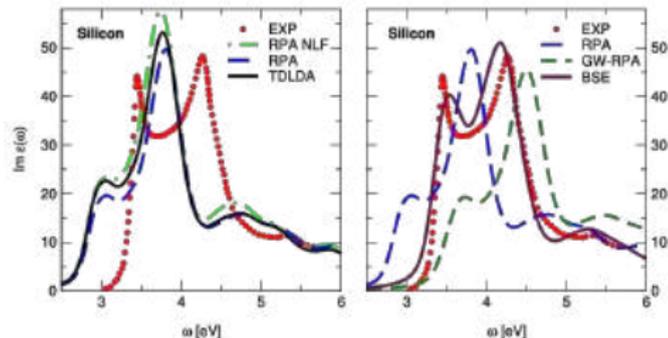


<sup>1</sup>Kundys *et al.*, *Nat. Mater.* **9**, 803 (2010).



# Modelling light-matter interaction

- Most light-matter *ab-initio* methods focus on describing optical absorption spectra.
  - Independent Particle Approximation (IPA)
  - Random Phase Approximation (RPA)
  - Time-Dependent Density Functional Theory (TD-DFT)
  - Bethe-Salpeter Equation (BSE)



- But difficult to relax structure!

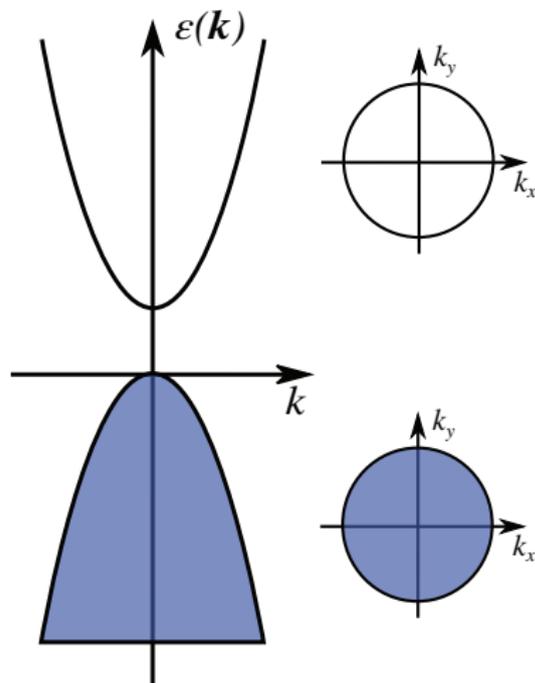
Olevani, *Structures on Different Time Scales*, vol. 1





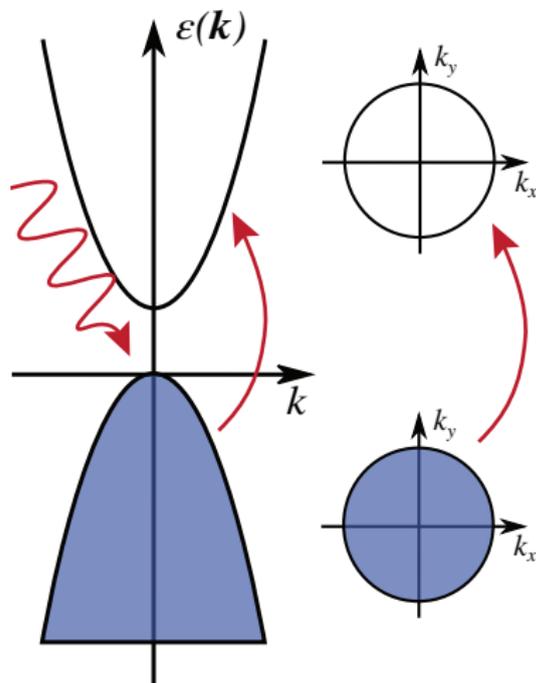
# Electronic processes during optical excitation

- Absorption of photon  $\hbar\omega > E_g$  and electron-hole pair creation.
- "Isotropification" of momenta through  $e - e$  elastic scattering.
- "Thermalization" (relaxation to band edges) through  $e$ -phonon inelastic scattering.
- Recombination (radiative or non-radiative), *i.e.* destruction of  $e - h$  pairs.



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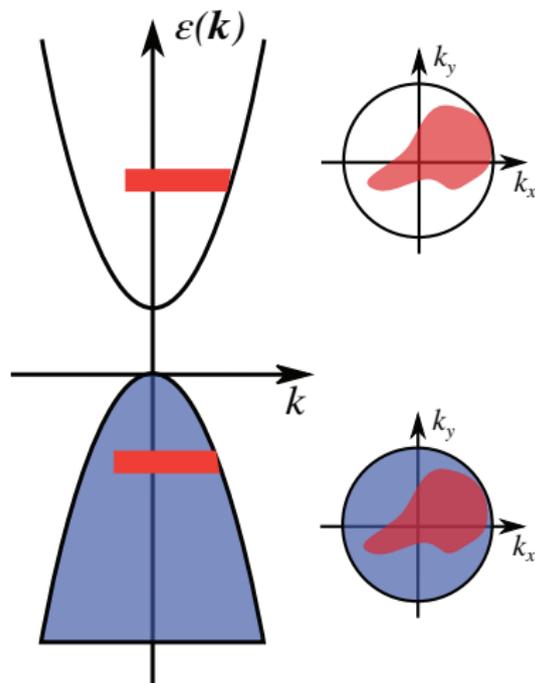
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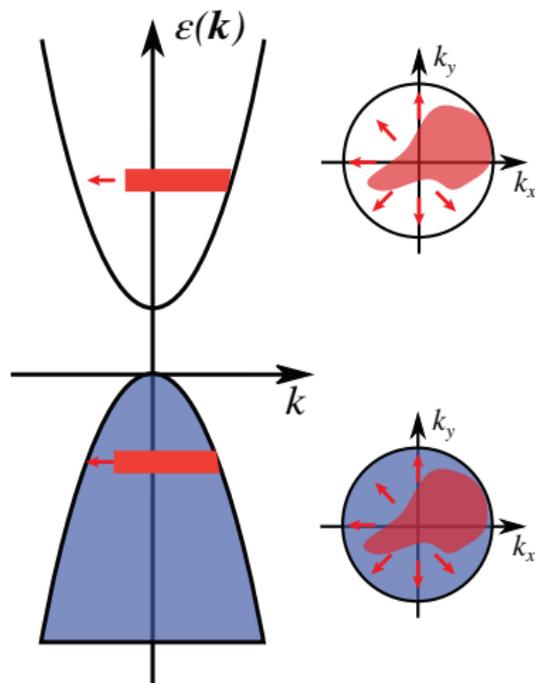
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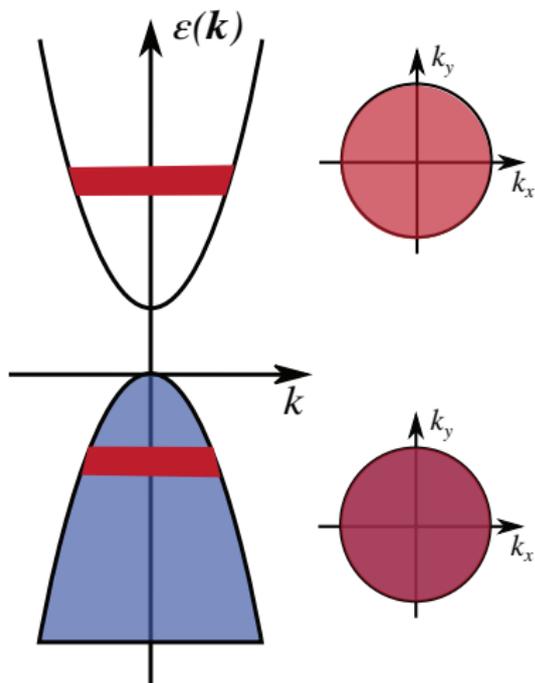
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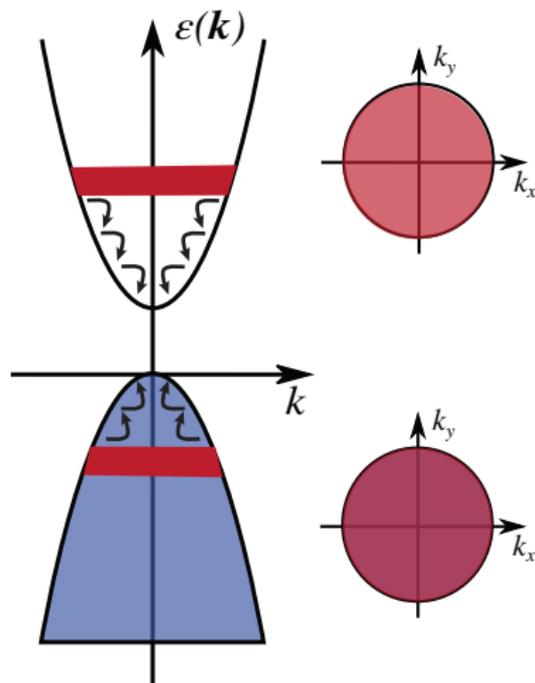
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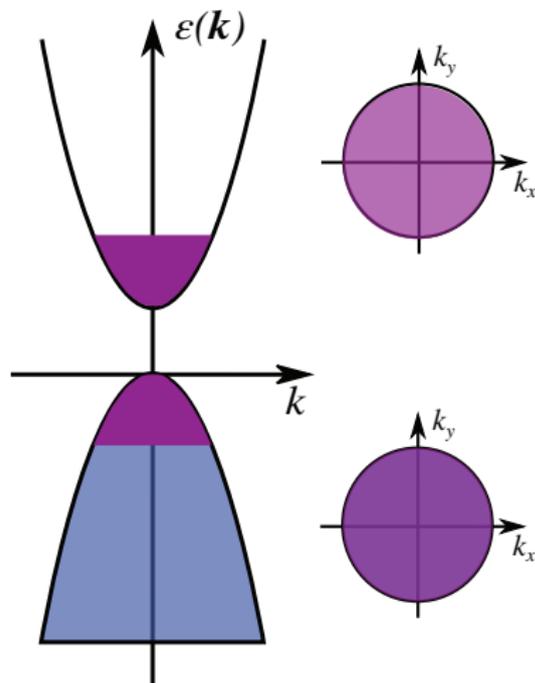
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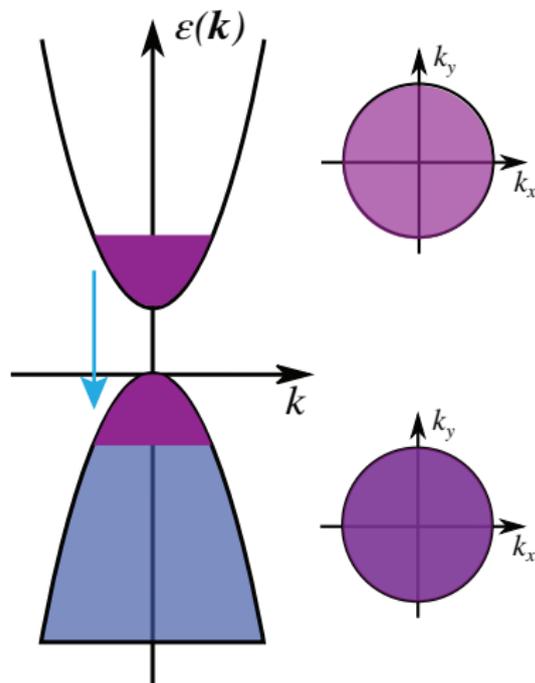
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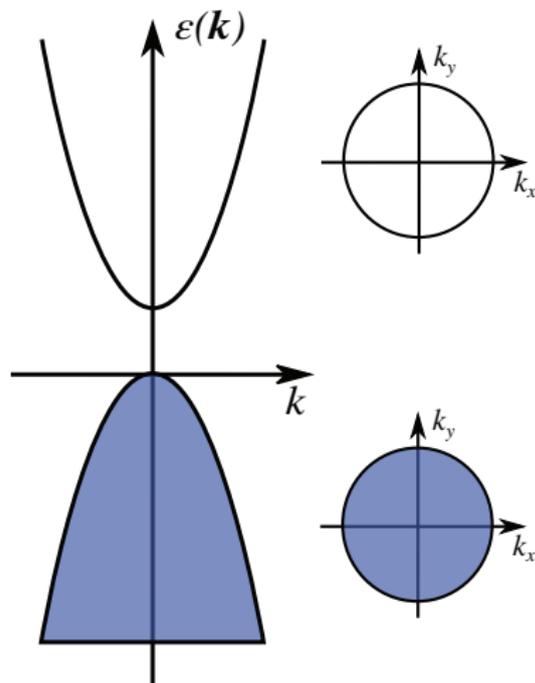
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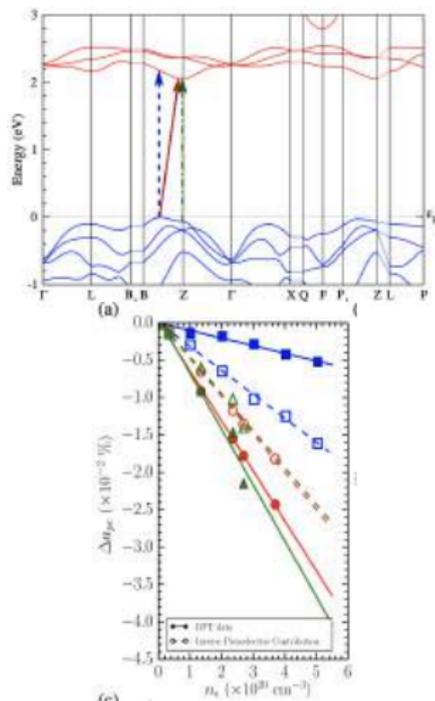
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# The $\Delta$ SCF method

- One possibility to relax structure while "mimicking" photo-excitation:  $\Delta$ SCF method.
- Basically, fix occupation number by hand. Already doable in abinit using `occpt 2` and `occ`.

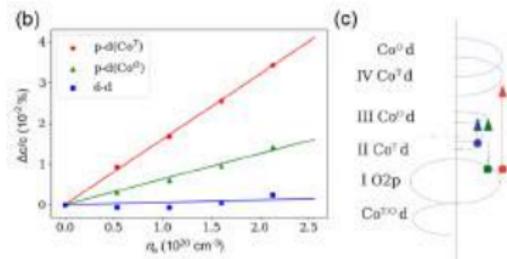
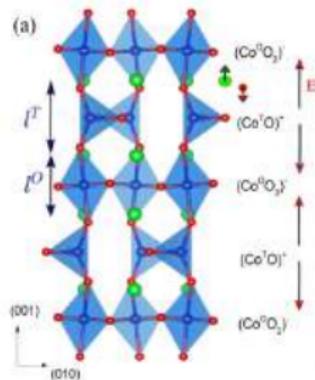


C. Paillard *et al.*, *Phys. Rev. Lett.* **116**, 247401 (2016).



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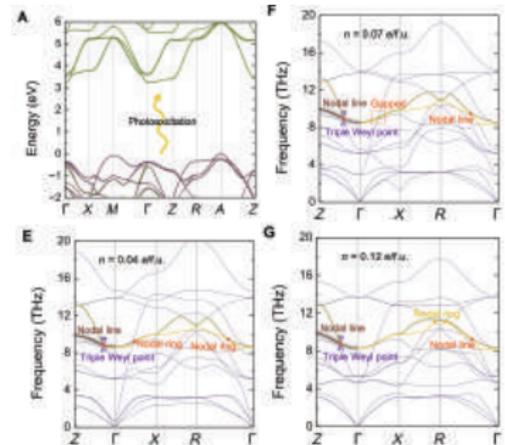


B. Zhang *et al.*, *Phys. Rev. B* **100**, 144201 (2019).



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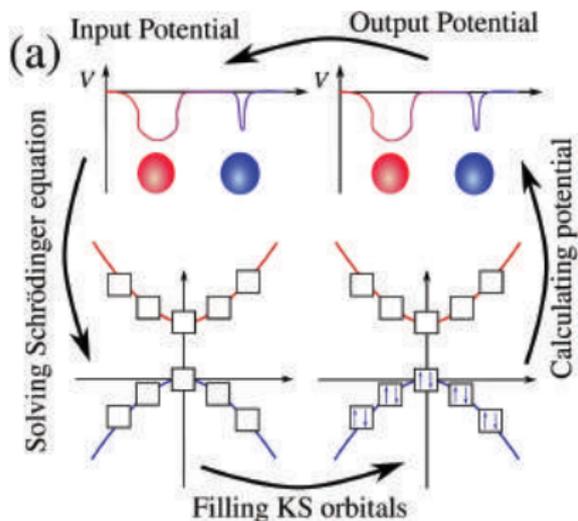


B. Peng *et al.*, *Sci. Adv.* **6**, eabd1618 (2020).

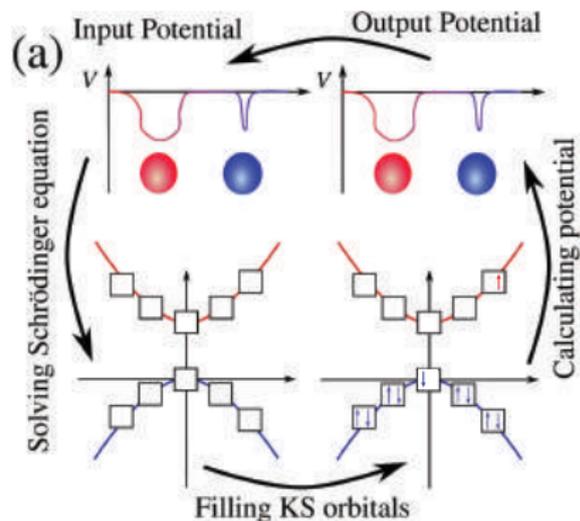


# The $\Delta$ SCF method

SCF cycle



$\Delta$ SCF cycle



Yang *et al.*, *J. Phys.: Cond. Matt.* **30**, 073001 (2018).





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- 2 **Implementation**
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# Implementation

## KS-DFT SCF cycle

- 1 Initialize potential from density
- 2 Diagonalize Hamiltonian and find Kohn-Sham orbitals and energies  $\varepsilon_{nk\sigma}$ .
- 3 Determine occupancy of orbitals: find  $\mu$  such

$$\sum_{n,\mathbf{k},\sigma} w_{\mathbf{k}} f(\varepsilon_{nk\sigma}, \mu, T_{smear}) = n_{elec}$$

using bi-section algorithm.

- 4 Deduce the electronic density. Go back to 1) if not converged.



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using bisection algorithm.

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# Implementation

KS "c"-DFT SCF cycle

- 1 Initialize potential from density
- 2 Diagonalize Hamiltonian and find Kohn-Sham orbitals and energies  $\varepsilon_{nk\sigma}$ .
- 3 Determine occupancy of orbitals: find  $\mu_e, \mu_h$  such

$$\sum_{n \in \text{conduction}, \mathbf{k}, \sigma} w_{\mathbf{k}} f(\varepsilon_{nk\sigma}, \mu_e, T_{\text{smear}}) = n_{ph}$$

AND

$$\sum_{n \in \text{valence}, \mathbf{k}, \sigma} w_{\mathbf{k}} f(\varepsilon_{nk\sigma}, \mu_h, T_{\text{smear}}) = n_{elec} - n_{ph}$$

using bisection algorithm.

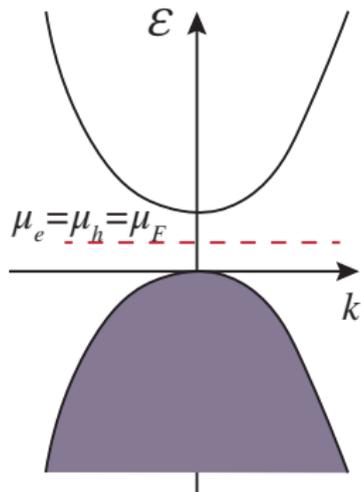
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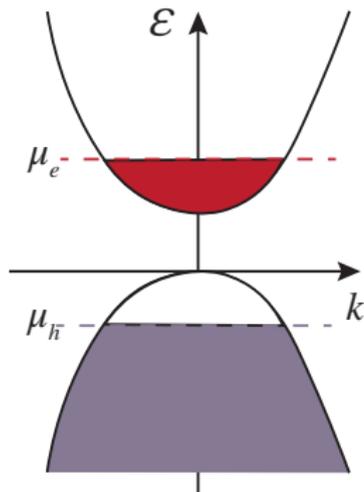
# Physical picture

$\mu_e$  and  $\mu_h \equiv$  quasi-equilibrium Fermi-Dirac chemical potentials for electrons in CB (holes in VB).

## Normal SCF



## constrained SCF





# Inputs

## Fermi-Dirac DFT

```
occopt 3  
tsmear 0.004
```

## Constrained DFT

```
occopt 9  
ival 88  
nqFD 0.10  
tsmear 0.004
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index last valence band  
number of excited  $e^-$  in CB and  $h^+$   
in VB



# Outputs

Fermi energy for thermalized electrons  
and holes (hartree) = 0.28525, 0.17012  
Average Vxc (hartree) = -0.34726  $\mu_e, \mu_h$

|     |          |          |          |          |          |          |
|-----|----------|----------|----------|----------|----------|----------|
| occ | 2.000000 | 2.000000 | 1.709670 | 1.286049 | 0.031578 | 0.000000 |
|     | 0.000000 | 0.000000 |          |          |          |          |
|     | 2.000000 | 2.000000 | 1.999997 | 1.984930 | 0.009216 | 0.000000 |
|     | 0.000000 | 0.000000 |          |          |          |          |
|     | 2.000000 | 2.000000 | 2.000000 | 1.999995 | 1.993200 | 0.000002 |
|     | 0.000000 | 0.000000 |          |          |          |          |
|     | 2.000000 | 2.000000 | 2.000000 | 1.999835 | 0.000003 | 0.000000 |
|     | 0.000000 | 0.000000 |          |          |          |          |
|     | 2.000000 | 2.000000 | 1.999998 | 0.013526 | 0.000012 | 0.000000 |
|     | 0.000000 | 0.000000 |          |          |          |          |
|     | 2.000000 | 2.000000 | 2.000000 | 1.999740 | 0.635282 | 0.000000 |
|     | 0.000000 | 0.000000 |          |          |          |          |
|     | 2.000000 | 2.000000 | 2.000000 | 2.000000 | 1.999974 | 0.207407 |
|     | 0.000000 | 0.000000 |          |          |          |          |
|     | 2.000000 | 2.000000 | 2.000000 | 2.000000 | 0.001207 | 0.000000 |
|     | 0.000000 | 0.000000 |          |          |          |          |
|     | 2.000000 | 1.983120 | 0.000027 | 0.000027 | 0.000247 | 0.000001 |
|     | 0.000001 | 0.000000 |          |          |          |          |
|     | 2.000000 | 2.000000 | 0.518049 | 0.518049 | 1.362480 | 0.000000 |
|     | 0.000000 | 0.000000 |          |          |          |          |





# What it can and can't do?

- ✓ SCF
- ✓ structural relaxation
- ✓ spin-orbit coupling
- ✗ DFPT
- ✗ Fixed magnetization (spinmagnetarget)



## Main modifications: 61\_occeig/m\_occ.F90

- `getnel`
  - Added `fermih`, `ibandmin`, `ibandmax` to the list of arguments
  - When `occopt=9`, computes the number of electron for bands located between `ibandmin` and `ibandmax`.
- `newocc`
  - added `fermih`, `ivalence`, `ne_qFD` and `nh_qFD`
  - `occopt=9`: determines `fermie` and `fermih` of electrons and holes with the constraints of having `nelec - nh_qFD` for bands 1 up to `ivalence` and `ne_qFD` for bands  $\geq \text{ivalence} + 1$

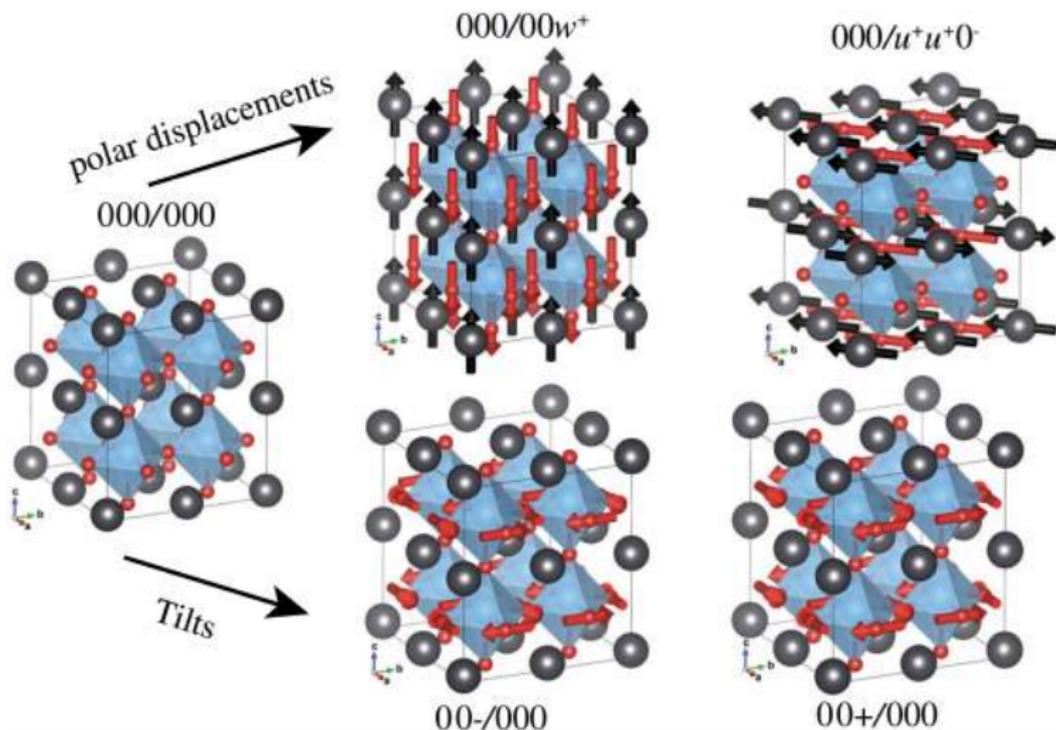


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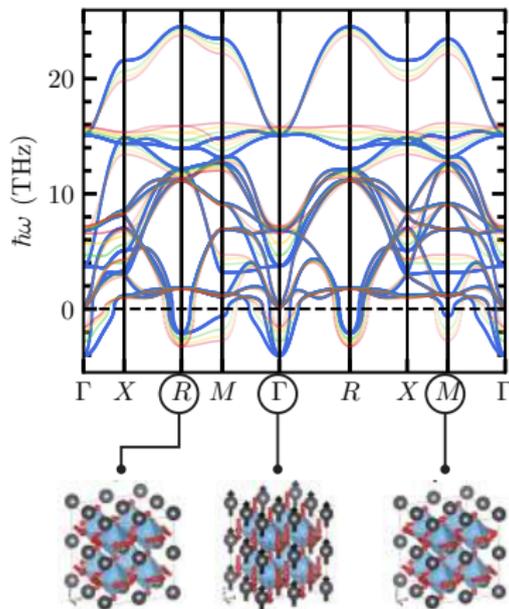
# Ferroelectric instabilities in perovskite oxides





# Ferroelectric instabilities in perovskite oxides

- Phonon in  $\text{PbTiO}_3$  cubic phase using PHONOPY
  - $2 \times 2 \times 2$  perovskite supercell (40 atoms)
  - $8 \times 8 \times 8$   $\Gamma$ -centered  $k$ -mesh.
  - PAW; 35 Ha plane wave cut-off.
- Increasing  $n_{ph} \Rightarrow$ 
  - $\Gamma$  (polar) instability "less" unstable.
  - $M$ -,  $R$ -instabilities (tilts) becomes dominant instability.

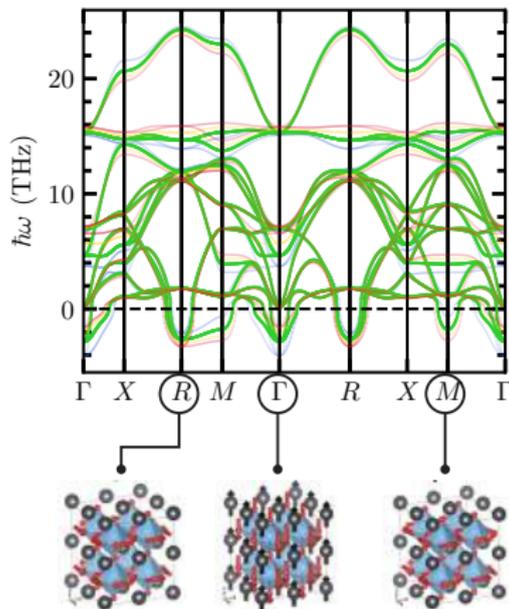


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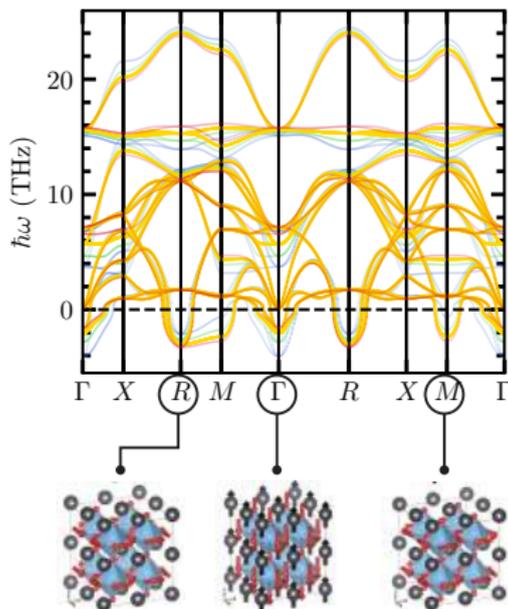
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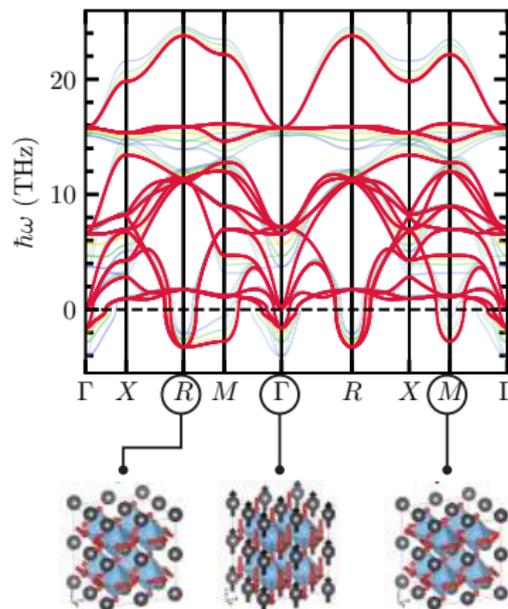
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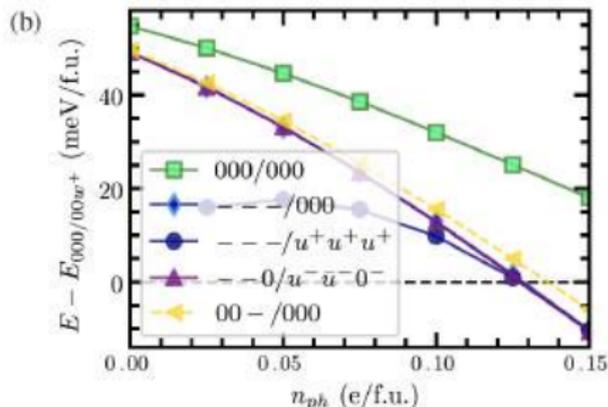


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# Photo-induced phase transition

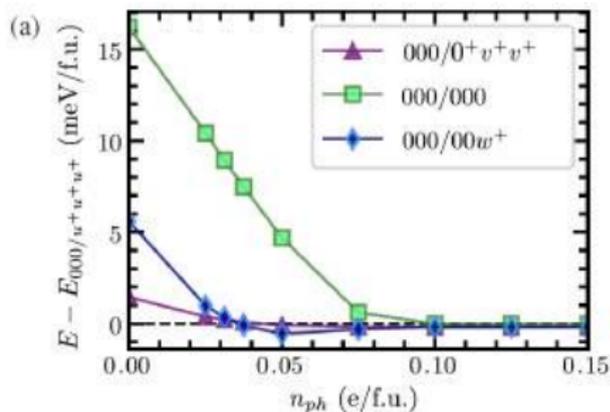
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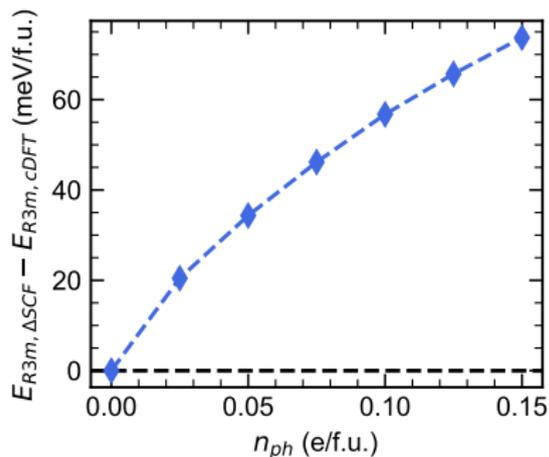
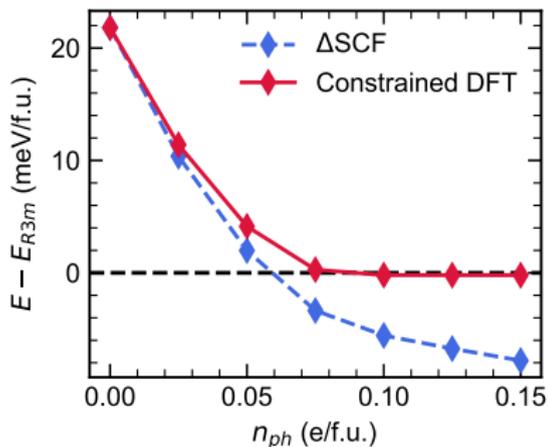
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# "constrained DFT" versus $\Delta$ SCF

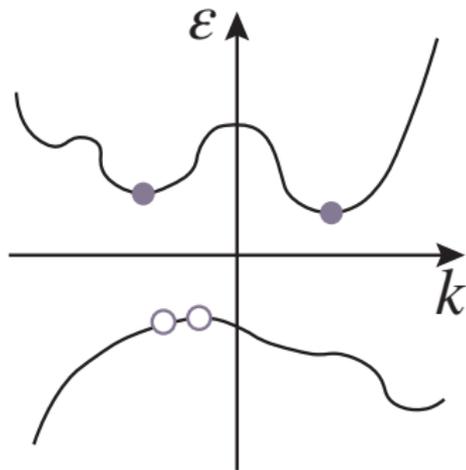


- BTO treated in  $\Delta$ SCF and c-SCF.
- Final structure are similar: cubic paraelectric.

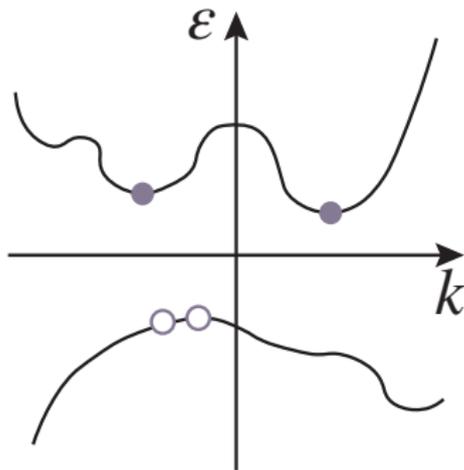


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constrained-DFT



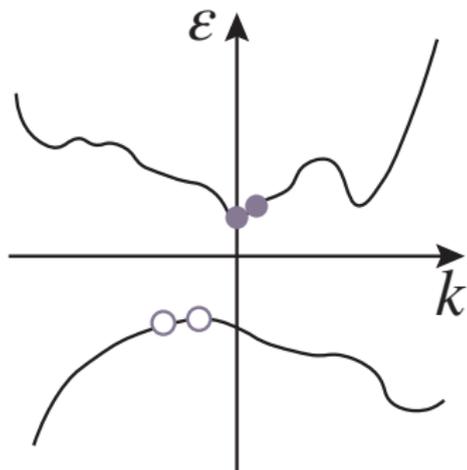
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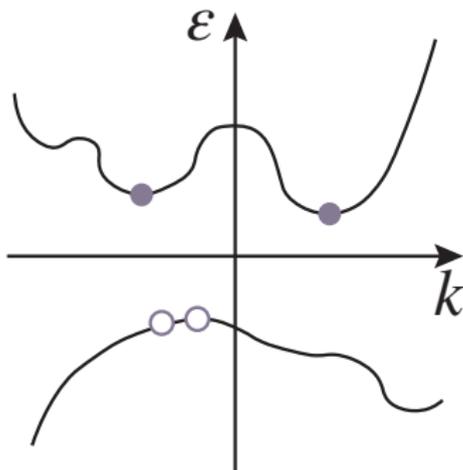


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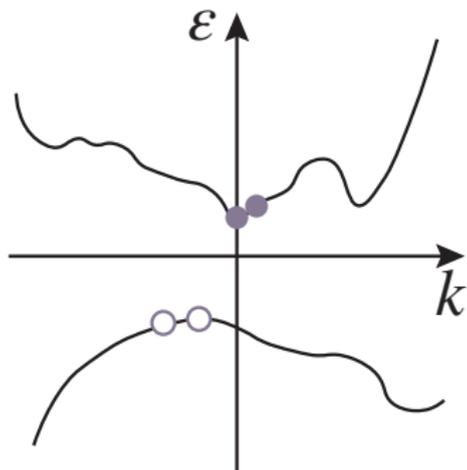
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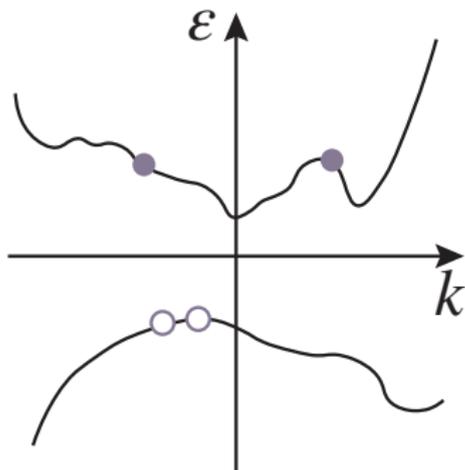


# "constrained DFT" versus $\Delta$ SCF

constrained-DFT



$\Delta$ SCF





# Summary

- Constrained occupation scheme: quasi-equilibrium
  - nqFD  $e^-$  in conduction bands with  $\mu_e$
  - nqFD  $h^+$  in valence bands with  $\mu_h$
- Input: `occopt 9, tsmear, ival,nqfd`
- Complementary to  $\Delta$ SCF method.
- Perspectives:
  - different concentrations of holes and electrons (charged system).
  - DFPT



# Thanks & Acknowledgements



Pr. Laurent Bellaïche

Jean-Michel Beuken

Pr. Éric Bousquet

# Thank you!



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